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SHORT COMMUNICATION

RECESSION RATES OF WATERFALLS IN BOSO PENINSULA, JAPAN, AND A PREDICTIVE EQUATION

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ABSTRACT

Rates of waterfall recession, and major factors that influence the rate, were studied using waterfalls in Boso Peninsula, Japan. The mean rate of waterfall recession was estimated by determining the age and original location. The principal factor in determining the rate of waterfall recession is the ratio of the erosive force of stream to the bedrock resistance. This is expressed in terms of measurable variables, which include the discharge (drainage area and precipitation), the width and height of the waterfall, and the unconfined compressive strength of the bedrock. An empirical equation connecting the force/resistance ratios and the rates of waterfall recession is derived. Copyright © 2003 John Wiley & Sons, Ltd.

KEY WORDS: waterfall; waterfall recession; rock control; erosion

INTRODUCTION

A waterfall, where water drops freely or flows almost vertically, is a typical form of knickpoint, or knickzone comprising a group of knickpoints. Modern studies suggest that recession of waterfalls or knickpoints plays an important role in fluvial systems (e.g. Young, 1985; Seidl *et al*., 1997; Righter, 1997; Heimsath *et al*., 2001; Niemann *et al*., 2001; Florsheim *et al*., 2001; Gonzalez, 2001; Downs and Simon, 2001; Zaprowski *et al*., 2001). Mechanisms by which waterfalls or knickpoints recede have been discussed based on field observations and laboratory experiments. Field observations include Niagara Falls in America (e.g. Gilbert, 1907; Philbrick, 1970, 1974), Kegon Falls in Japan (Mino, 1958), some examples from southeastern Australia (Young, 1985), and 43 waterfalls in the Outer Carpathians in Poland (Alexandrowicz, 1994). Laboratory experiments have been performed by Holland and Pickup (1976), Gardner (1983), Mathewson (1989), May and Mathewson (1989) and others. However, the recession mechanism is still incompletely understood. The rate of waterfall recession was 1·2–2·0 m a^{-1} for Niagara Falls from 1842 to 1905 (Gilbert, 1907), 0·09–0·15 m a^{-1} for Victoria Falls in Zambia (Derricourt, 1976) and 1·0–2·0 m a^{-1} for Ryumon Falls in Tochigi Prefecture, Japan (Yoshida and Ikeda, 1999). The factors influencing the rate of waterfall recession are not settled. Considering factors expected to be significant to estimate the recessional rates of specific waterfalls, the present study reveals the quantitative relation between the rates of waterfall recession and these factors.

STUDY SITES

The Boso Peninsula area in Japan (Figure 1) has a temperate humid climate with a mean annual air temperature of about 15 °C and mean annual precipitation of 1300–2000 mm (Japan Meteorological Agency, 1991). In

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this area: (1) more than 170 waterfalls have been formed as a result of the high rates of tectonic uplift and fluvial incision; (2) most such waterfalls are small (up to 40 m height) and easy to work with; and (3) rates of landform change are generally high because of the weak geology comprising mainly Neogene or Quaternary sedimentary rocks. The Boso Peninsula has had a seismic uplift trend during the Quaternary, of which rates become higher southward, causing many rivers to exhibit a deeply incised meander, and extensive uplifted coastal and river terraces have developed in the southern area. Detailed studies of such terraces (e.g. Nakagawa, 1977; Nakata *et al*., 1980; Kashima, 1982) allow dating of the formative ages of the terraces.

We selected nine waterfalls (Figure 1) for which the rate of recession is easily determined. None of the waterfalls has any unusual geological characteristics, such as the caprock structure at Niagara Falls, and they do not overhang, although they have steep faces that are nearly vertical. Most of the water flow is rapid and down the faces of the waterfalls.

ESTIMATION OF RATES OF WATERFALL RECESSION

Actual recession of the waterfalls under study can hardly be observed, because the rates are too small. These waterfalls have clearly recessed, however, since gorges or steep riverside cliffs are found downstream of the present locations of many (Figure 2), although the vertical cliffs of some of such gorges seem to have declined due to denudation. By supposing that waterfalls have recessed gradually and are continuing to recede, mean rates of waterfall recession from their time of origin to the present can be estimated.

The present location is apparent, so that the distance of recession, D, is estimated by determining the location at which the waterfall originated. On the assumption that parallel recession has taken place, the present location of the waterfall is regarded as that of the baseline of waterfall face. The duration of recession, T, is also

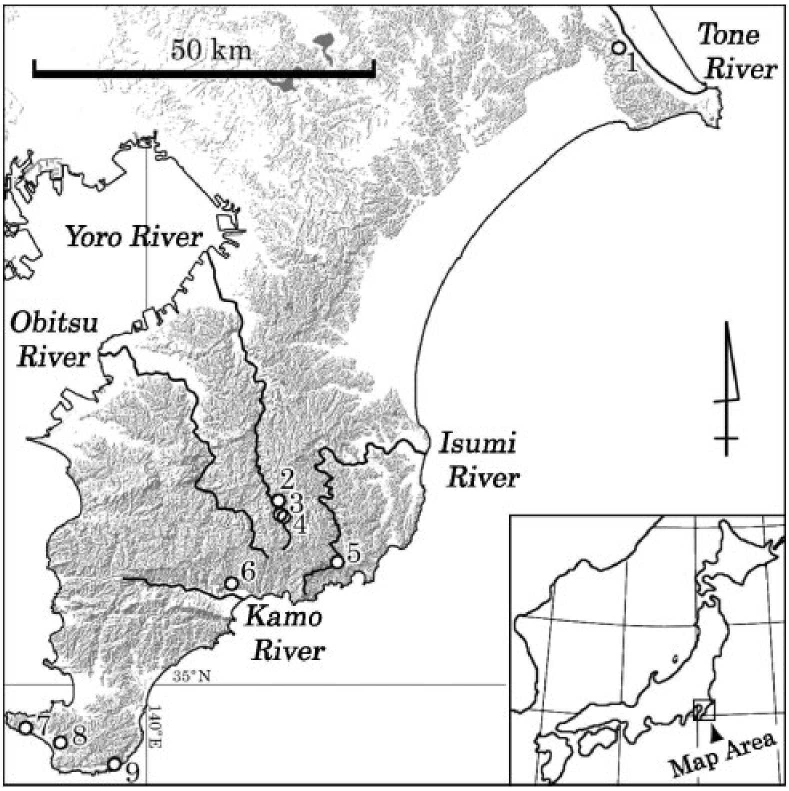

Figure 1. Study area. Numbers show locations of studied waterfalls (see Table I)

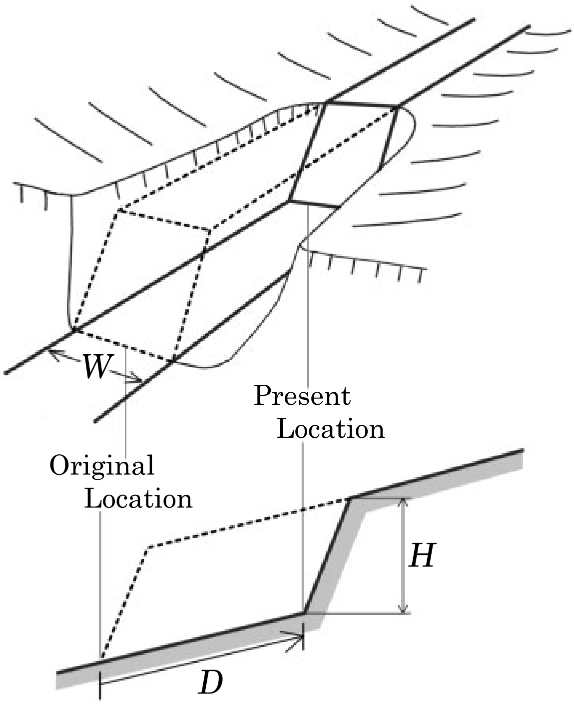

Figure 2. Model of waterfall recession. The dashed line represents the route of recession

estimated by determining the age of the waterfall. The mean rate of recession is then *D/T*. Estimation of *D* and *T* is undertaken for the nine waterfalls, which are classified into five types according to their origins.

Type A: waterfall originating with differing incision rates between mainstream and tributary

When the incision rate of a mainstream is far higher than that of a tributary, a waterfall can form at the confluence (Figure 3). It is supposed that the waterfall begins to recess up the tributary after the incision stage of the mainstream ends. The origin of this type of waterfall, named Type A, is the confluence of the tributary and mainstream, so that the distance of recession is easy to estimate. The age of the waterfall is obtained from the age of formation of the riverside terrace; the high rate of incision of the mainstream not only causes a waterfall on a tributary, but may also leave the old floodplain as a terrace. The age of formation of the terrace represents the beginning of the incision stage. According to Suzuki *et al*. 1983, the incision stage is very short in comparison with the lateral erosion stage, so that the duration of the incision stage can essentially be ignored. The age of formation of the terrace is therefore taken to coincide with the cessation of incision. Since the ending of the incision stage represents the beginning of waterfall recession, the age of the waterfall is obtained from the age of formation of the river terraces.

Type B: waterfall originating with uplifted sea-cliff

Waterfalls originating by means of an uplifted sea-cliff are named Type B (Figure 4). The waterfall is supposed to have originated with sea-cliff formation by coastal waves at a time of higher sea level, and to have begun recessing when the shoreline emerged via a crustal seismic uplift. By supposing that the coastal waves undercut the original sea-cliff giving rise to a rate of parallel recession that exceeds the rate of waterfall recession, the shoreline would then be smooth in shape, and the waterfall would fall directly into the sea. The location of waterfalls can therefore be estimated by restoring previous shorelines smoothly, as shown in Figure 4. The time at which waterfall recession begins is supposed to be the same as the time at which uplifted shore platforms form in front of the waterfall.

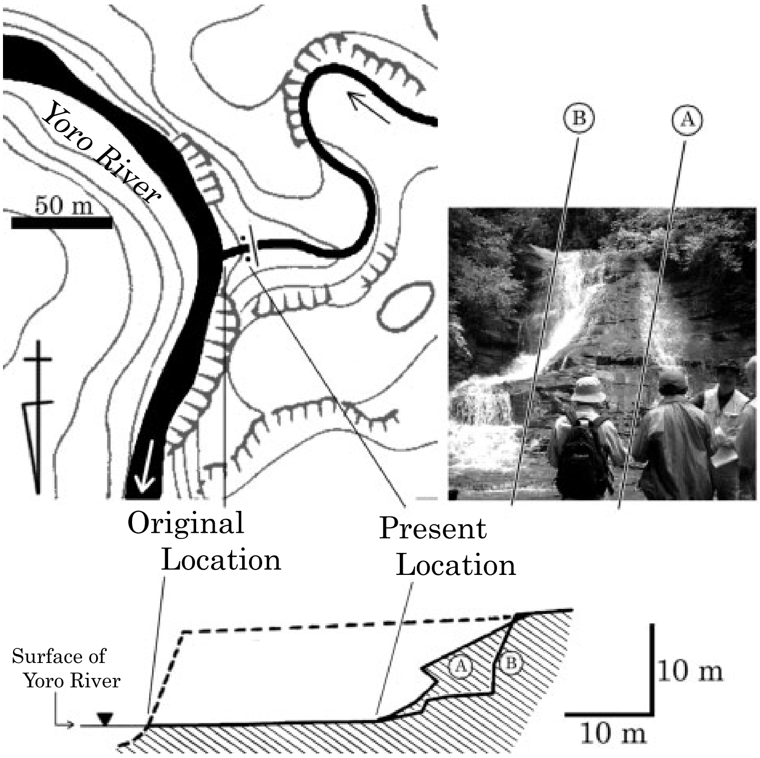

Figure 3. Plan view, longitudinal profile and photographs of Takisawa-no Falls, as an example of Type A. The profile of the waterfall is measured along two lines (A and B), as represented in the photograph. All Type A waterfalls studied here are located in the middle part of the Yoro River. The erosive terrace surfaces in this area are classified into seven categories, from the highest (70–80 m high) to the lowest (about 15 m high) (Kashima, 1982). All of the Type A waterfalls studied here are slightly below the height of the lowest surfaces, and the waterfalls are supposed to have formed along with the lowest surfaces. The third lowest surfaces are known to be 4000–5000 years old, and are about 45 m high in this area (Kashima, 1982), so that, supposing that the uplift rate is almost constant, the lowest surfaces are approximately 1500 years old, and so are these waterfalls. In the case of Takisawa-no Falls, the rate of recession is calculated to be 24·0 m/1500 years = 0·016 m a^{-1}

Other types of waterfalls

There are three other types of waterfalls: Type C–abandoned waterfall which has been maintained artificially; Type D–waterfall originating with artificial shortcut; Type E–waterfall originating with emergence of buried sea-cliff.

Soho Falls, an example of a Type C waterfall, is just 0·2 m in width and 1·8 m high; it was used as a bathing place, perhaps for religious purposes, and the waterfall was artificially maintained until approximately 50 years ago. It is assumed that the waterfall did not recess during artificial maintenance since a drainpipe would have been placed on it, and that it began to recess as soon as it ceased to be used (Yoshimura, 2001).

The artificial bypassing of an incised meandered, soft-bedrock channel, called 'kawa-mawashi' in southern Boso district, often resulted in waterfalls classified as Type D. The location of the Type D waterfall is usually taken as the outlet point of the new channel. The age can be settled from the historical record of the kawa-mawashi construction, although many records no longer exist or are unavailable. An example of Type D is Afuri Falls, of which kawa-mawashi construction is unusual in that flow continues in the original channel, which today takes about one-third of the water.

We include a waterfall that is apparently formed as a result of differing rates of incision, caused probably by crustal uplift between the lower alluvial deposits and the upper bedrock on a riverbed; the interface is a re-emerged sea-cliff that had been buried at a time of extremely high sea level. The original location of the waterfall is a border of alluvial deposits and bedrock on the riverbed downstream from the present waterfall.

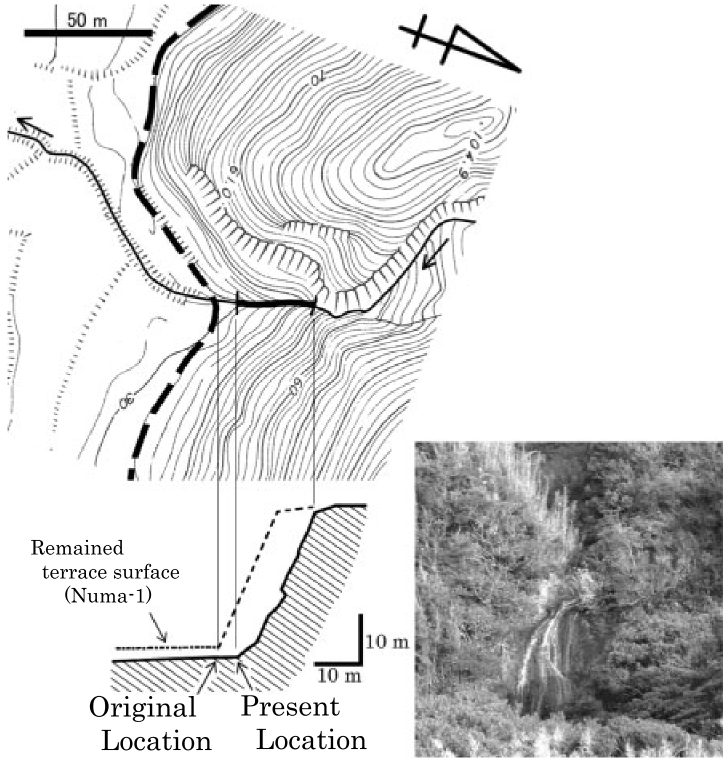

Figure 4. Photograph, plan view and longitudinal profile of Ito-fudo Falls as an example of Type B. The waterfall is supposed to have begun recessing when the Numa-1 surface had formed, 6150 years ago (Nakata *et al*., 1980), and the past shoreline is restored as the dashed line. The rate of recession of Ito-fudo Falls is calculated to be 0·0013 m a^{-1}

Since Zenzen Falls, an example of a Type E waterfall, seems to have recessed leaving terrace surfaces on the riverside, the age of the waterfall can be obtained from that of the lowest terrace surfaces, which is estimated at 2000 years (Maeda, 1995).

All results are shown in Table I. The Sanogawa-no Falls (Type B) has the highest recession rate (0·27 m a^{-1}), and the Ito-fudo Falls (Type B) has the lowest rate (0·0013 m a^{-1}). The mean value is approximately 0·09 m a^{-1}.

Table I. Estimated values of D, T and D/T, with other basic information about geology

Location (see Figure 1)	Name of waterfall	Type of origin	Dominant lithology	Geological era	Distance of recession D (m)	Duration of recession T (years)	Rate of recession D/T (m a^{-1})
1	Soho Falls	C	Mudstone	Pleistocene	6·4	50	0·13
2	Oikawa-fudo Falls	A	Mudstone	Pliocene	100	1500	0·067
3	Fukasawa-no Falls	A	Mudstone	Pliocene	26·5	1500	0·018
4	Takisawa-no Falls	A	Mudstone	Pliocene	24·0	1500	0·016
5	Afuri Falls	D	Mudstone	Pliocene	55	300	0·18
6	Zenzen Falls	E	Mudstone	Miocene	200	2000	0·10
7	Ito-fudo Falls	B	Mudstone	Miocene	8	6150	0·0013
8	Sanogawa-no Fall	B	Mudstone	Pliocene	80	299	0·27
9	Ryogenji Falls	B	Mudstone	Miocene	11	6150	0·0018

THEORETICAL BACKGROUND

Waterfall recession takes place because the erosive force, deriving from streaming water that includes transported debris, operates on the part of the waterfall face in the bedrock. The ratio of the erosive force, F, to the bedrock resistance, R, is therefore a good measure of the rate of waterfall recession. To study the rate of waterfall recession, the force/resistance ratio, expressed here as the index F/R, is expressed in terms of measurable parameters as described below.

The erosive force per unit area, F, clearly increases with the amount of discharge and decreases with the area of waterfall face suffering erosion. The mass of water depends on the density of water, ρ, and the annual discharge, Q while the discharge depends on the drainage area upstream of the waterfall, A, and the annual precipitation in the catchment area, P. Since the data of the long-term discharge are not available, the product of A and P is used here as the counterpart of the discharge. The area of the waterfall face is approximately WH, since the face is nearly vertical. The erosive force F can therefore be modelled as:

$$F \propto (\rho, A, P, W, H) \tag{1}$$

The effect of transported debris in the stream has been ignored, since it is difficult to measure.

The unconfined compressive strength, S_c, is considered to be the most effective single parameter expressing bedrock resistance. Ignoring discontinuities, strength anisotropy, etc., the bedrock resistance per unit area R is therefore modelled by:

$$R \propto (S_c) \tag{2}$$

Dimensional analysis on these physical quantities, grouped as ρ, AP, WH, and S_c, indicates that the dimensionless index F/R takes the form:

$$\frac{F}{R} = \frac{AP}{WH}\sqrt{\frac{\rho}{S_c}} \tag{3}$$

MEASUREMENT OF PARAMETERS

To evaluate the F/R index, we made measurements of the parameters involved. The drainage area upstream of the waterfall, A, was measured from the topographic map of scale of 1 : 25 000 published by the Geographical Survey Institute. The mean annual precipitation of the drainage basin, P, was obtained from the distribution of precipitation held by Chiba Prefecture Civil Engineering Department 1999. The width, W, and height, H, were obtained by field measurement. The density of water ρ is taken as 1×10^3 kg m^{-3}. We measured the Schmidt rock hammer rebound value of the bedrock, R_s, at and around the waterfall under wet conditions. The rebound value R_s was transformed into the unconfined compressive strength value, S_c, using the graph given by Hoek and Bray (1981, p. 98), where the unit weight of wet rock was taken as 20 kN m^{-3} according to Matsukura and Yatsu's 1982 and Matsukura and Matsuoka's 1996 data from the studied area. Afuri Falls, which takes only two-thirds of the flow in that river as stated above, has had its value of AP corrected by this factor. The data thus obtained and the calculated values of the index F/R are summarized in Table II. It is apparent that Soho Falls has the highest value of F/R ($3{\cdot}51 \times 10^{-4}$) and Ito-fudo Falls has the lowest value ($0{\cdot}23 \times 10^{-6}$).

ANALYSIS AND DISCUSSION

Figure 5 shows plots of the index F/R versus the mean rate of waterfall recession D/T for each waterfall. The best-fit line through the data points in Figure 5 is given by:

$$\frac{D}{T} = 99{\cdot}7\left[\frac{AP}{WH}\sqrt{\frac{\rho}{S_c}}\right]^{0{\cdot}73} \tag{4}$$

where the coefficient value of 99·7 holds an identical dimension as that of D/T (m a^{-1}).

Table II. Measured parameters and calculated value of *F/R* index

Location and name of waterfall	Drainage area A ($\times10^6$ m^2)	Mean annual precipitation P (mm a^{-1})	Mean annual precipitation P ($\times10^{-8}$ m s^{-1})	Counterpart of discharge AP ($\times10^{-2}$ m^3 s^{-1})	Width W (m)	Height H (m)	Schmidt rock hammer rebound value R_S (%)	Unconfined compressive strength S_C ($\times10^6$ N m^{-2})	Calculated *F/R* index (see Eqn 3)
1 Soho Falls	0·29	1600	(5·1)	1·5	0·17	1·8	14·5	18·1	3.51×10^{-4}
2 Oikawa-fudo Falls	2·0	2000	(6·3)	13	6	6·1	17	20·1	2.45×10^{-5}
3 Fukasawa-no Falls	0·60	2000	(6·3)	3·8	6	11	32	37·2	2.97×10^{-6}
4 Takisawa-no Falls	1·1	2000	(6·3)	7·0	8	12	32	37·2	3.77×10^{-6}
5 Afuri Falls	14	2000	(6·3)	91	8·6	3·6	15·5	18·9	2.14×10^{-4}
6 Zenzen Falls	11	2000	(6·3)	69	32	5	14	17·8	3.24×10^{-5}
7 Ito-fudo Falls	0·18	1800	(5·7)	1·0	9	32	20·5	23·2	2.34×10^{-7}
8 Sanogawa-no Falls	3·6	1800	(5·7)	21	2	4·4	10·5	15·4	1.88×10^{-4}
9 Ryogenji Falls	0·34	1800	(5·7)	1·9	4	30	21	23·7	1.05×10^{-6}

This relation indicates that the rate of waterfall recession increases as the *F/R* index increases. The rate of waterfall recession is approximately explained using only the parameters on the right-hand side of Equation 4 (drainage area, precipitation, fall width, height and bedrock strength) without considering other factors such as the mechanism of erosion on waterfalls.

The rate of recession of Ryumon Falls in Tochigi Prefecture, Japan, has already been reported. This waterfall is in E-gawa River, which is a tributary of the Naka River, and in view of its origin it can be classified here as Type A. Yoshida and Ikeda 1999 estimated the rate of recession of this waterfall as 0·1–0·2 m a^{-1}, supposing that the falls originated at the confluence of the E-gawa River and Naka River. These authors gave the drainage area, the width and height of the waterfall, and the unconfined compressive strength, and since the value of the mean annual precipitation of the catchment is available from *The Climatic Atlas of Japan* (Japan Meteorological Agency, 1971), all values on the right-hand side of Equation 4 can be calculated. The drainage area $A = 90$ km^2, the width of the waterfall $W = 65$ m, the height $H = 12$ m, the mean annual precipitation $P = 1300$ mm a^{-1}, and the unconfined compressive strength $S_c = 10–20$ MN m^{-2}. Equation 4 now predicts the rate of recession *D/T* to be 0·058–0·075 m a^{-1}. This is of the same order as the value 0·1–0·2 m a^{-1} estimated by Yoshida and Ikeda 1999, suggesting that Equation 4 is more widely applicable.

CONCLUSIONS AND ASSIGNMENTS

No previous studies have set out to clarify the factors that influence the rate of waterfall recession. Applying several measurable parameters, we investigated the relation between the ratio of the erosive force of stream to the bedrock resistance and the rate of waterfall recession. An empirical equation is derived. This equation is likely to be applicable to other waterfalls, as the example of Ryumon Falls has shown.

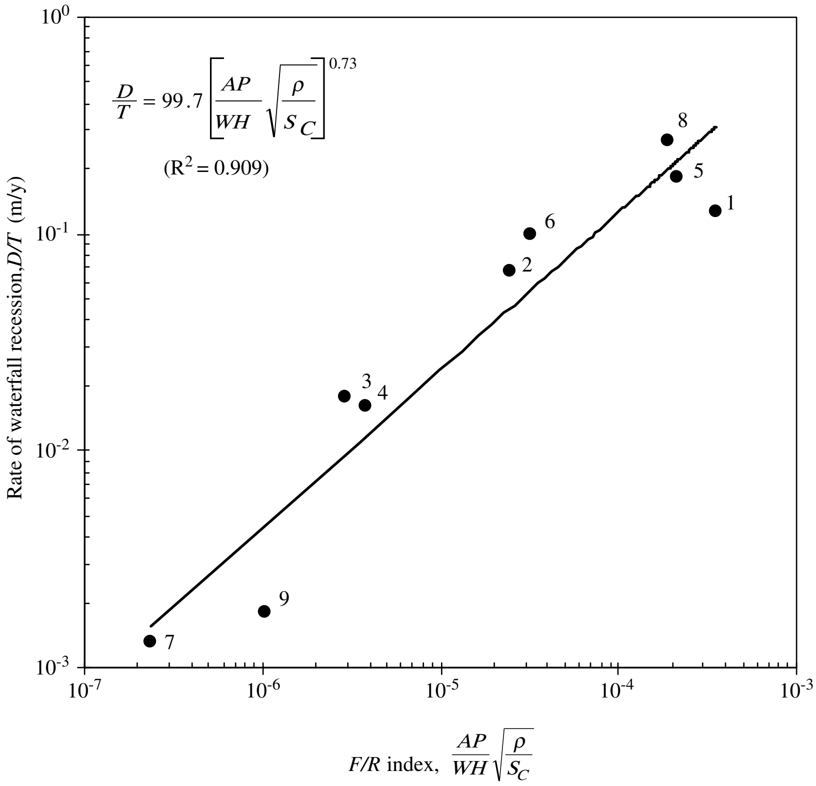

Figure 5. Relationships between the *F/R* index and the rates of waterfall recession

Since the analysis is still a crude approximation, it is difficult to have more detailed discussion at this stage. To improve this empirical equation: (1) other properties of bedrock besides its unconfined compressive strength should be taken into account, including discontinuities; (2) further factors affecting waterfall recession such as the amount of transported debris in the stream as an attrition material and gradient of waterfall face should be considered; (3) better ways of estimating the distance and the duration of waterfall recession should be sought; and (4) more data should be gathered on waterfalls in Boso Peninsula or other areas to test Equation 4. Together with the present results, studies of the rates of incision and lateral planation will improve understanding of the development of fluvial landforms.

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